Demonstration of quantum telecloning of optical coherent states

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We demonstrate unconditional telecloning for the first time. In particular, we symmetrically and unconditionally teleclone coherent states of light from one sender to two receivers, achieving a fidelity for each clone of $F = 0.58 \pm 0.01$, which surpasses the classical limit. This is a manipulation of a new type of multipartite entanglement whose nature is neither purely bipartite nor purely tripartite.

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Quantum telecloning [1] is a quantum information protocol combining cloning and teleporting into a single new primitive. The protocol offers significant technical advantages over the two-step local cloning plus teleportation strategy. In particular, in the case of coherent state telecloning, only finite entanglement is required for generating remote clones with optimal fidelity [2].

In fact, quantum telecloning generalizes quantum teleportation with multiple receivers [1]. In quantum teleportation, bipartite entanglement shared by two parties (Alice and Bob) enables them to teleport an unknown quantum state from Alice to Bob by communicating only through classical channels [3]. If three parties (Alice, Bob, and Claire) share an appropriate tripartite entangled state, Alice is able to teleport an unknown quantum state to Bob and Claire simultaneously. This is called "$1 \to 2$ quantum telecloning." More generally, quantum telecloning to an arbitrary number of receivers ($1 \to n$ quantum telecloning) can be performed by using multipartite entanglement.

The heart of quantum telecloning is the multipartite entanglement shared among the sender and receivers. Without multipartite entanglement, only the corresponding two-step protocol is possible: first the sender makes clones locally [4], and then sends them to each receiver with bipartite quantum teleportation [5] (or vice versa, teleporting followed by local cloning). The two-step protocol would require maximal bipartite entanglement for optimal fidelity teleportation (which for continuous-variable teleportation corresponds to states with infinite energy). Surprisingly, continuous-variable telecloning of coherent states requires only finite squeezing to achieve the same optimal fidelity [2]. In fact, the level of squeezing needed for optimal telecloning of coherent states is close to the reach of current technology [6].

We demonstrate the quantum telecloning of coherent states. The ability to reliably manipulate coherent states is of particular relevance to cryptographic scenarios. Indeed, in all non-heralded or non-entangled quantum cryptosystems the states used are invariably coherent states. More generally, experimental quantum telecloning provides us with another way of manipulating multipartite entanglement, a resource which plays an essential role in quantum computation and multiparty quantum communication. A related scheme for so-called partial teleportation involves one local and one remote clone. This scheme was demonstrated for photonic qubits [7], but could presumably be extended to coherent states.

Here the quantum state to be telecloned is that of an electromagnetic field mode. We use the Heisenberg picture to describe the evolution of the quantum state. An electromagnetic field mode is represented by an annihilation operator $\hat{a}$ whose real and imaginary parts ($\hat{a} = \hat{x} + i\hat{p}$) corresponds to the position and momentum quadrature-phase amplitude operators. These operators $\hat{x}$ and $\hat{p}$ satisfy the commutation relation $[\hat{x}, \hat{p}] = \frac{i}{2}$ (in so-called photon-number units with $\hbar = \frac{1}{2}$) and can be treated as canonically conjugate variables. This continuous-variable approach has attracted much interest because of the relative ease of realization of unconditional or deterministic quantum information processing [8]. Unconditional quantum teleportation was demonstrated for the first time with this approach [5], and various successful experiments have been reported [4, 5, 9, 10, 11, 12, 13, 14].

Quantum telecloning relies on tripartite entanglement — the minimum unit of multipartite entanglement. Tripartite entanglement for continuous variables can be generated by using squeezed vacuum states and two beam splitters [15]. Even infinitesimal squeezing can yield fully inseparable tripartite state [16]. The states so generated can be classified by the separability of the reduced bipartite state after tracing out one of the three subsystems. In the qubit regime, this classification is well established. For example, the Greenberger-Horne-Zeilinger (GHZ) state [17] does not have any bipartite entanglement after the trace-out, while the W state [18] does. In the continuous-variable regime, various types of tripartite entanglement can be generated by choosing proper transmittances/reflectivities of beam splitters and the levels

of squeezing. For example, the continuous-variable analogue of the GHZ state [13, 15] was used in the quantum teleportation network. This state can be created by combining three squeezed vacuum states with a high level of squeezing on two beam splitters, and is a tripartite maximally entangled state in the limit of infinite squeezing. In the absence of any bipartite entanglement between any pair of the three parties, quantum teleportation from a sender to a receiver cannot be achieved without the help of the third member. In contrast, the entanglement required for quantum telecloning comprises both a bipartite and a tripartite structure much like the W state [18], and it is not maximally entangled. We create this new type of tripartite entanglement and use it to demonstrate $1 \rightarrow 2$ quantum telecloning of coherent states.

The scheme for creating the tripartite entanglement for quantum telecloning [2] is shown in the center of Fig. 1. Two optical parametric oscillators ($\mathrm{OPO}_i, \mathrm{OPO}_{ii}$) pumped below oscillation threshold create two individual squeezed vacuum modes ($\hat{x}_i,\ \hat{p}_i$) and ($\hat{x}_{ii},\ \hat{p}_{ii}$). These beams are first combined with a 50/50 beam splitter with a $\pi/2$ phase shift and then one of the output beams is divided into two beams (B,C) with another 50/50 beam splitter. The three output modes ($\hat{x}_j,\ \hat{p}_j$) ($j = \mathrm{A,B,C}$) (abbreviated as ($\hat{x}_{\mathrm{A,B,C}},\ \hat{p}_{\mathrm{A,B,C}}$) hereafter) are entangled with arbitrary levels of squeezing.

Here, modes A and B and modes A and C are bipartitely entangled and modes A, B, and C are tripartitely entangled. On its own each mode is in a thermal state and shows excess noises. This can be verified by applying the sufficient inseparability criteria for a bipartite case [19, 20] and a tripartite case [16]. In the present situation, the criteria are

$$\begin{aligned}&\langle[\Delta(\hat{x}_{\mathrm{A}} - \hat{x}_{\mathrm{B,C}})]^2\rangle + \langle[\Delta(\hat{p}_{\mathrm{A}} + \hat{p}_{\mathrm{B,C}})]^2\rangle \\ &= \left(\frac{1-\sqrt{2}}{2}\right)^2 \left[\langle(\Delta\hat{x}_i)^2\rangle + \langle(\Delta\hat{p}_{ii})^2\rangle\right] \\ &+ \left(\frac{1+\sqrt{2}}{2}\right)^2 \left[\langle(\Delta\hat{x}_{ii})^2\rangle + \langle(\Delta\hat{p}_i)^2\rangle\right] + \frac{1}{4} < 1\ , \qquad (1)\end{aligned}$$

where $\langle(\Delta\hat{x}^{(0)})^2\rangle = \langle(\Delta\hat{p}^{(0)})^2\rangle = \frac{1}{4}$ and superscript (0) denotes a vacuum. The left-hand-side of the inequality can be minimized when $(\hat{x}_i, \hat{p}_i) = (e^r\hat{x}_i^{(0)}, e^{-r}\hat{p}_i^{(0)})$, $(\hat{x}_{ii}, \hat{p}_{ii}) = (e^{-r}\hat{x}_{ii}^{(0)}, e^r\hat{p}_{ii}^{(0)})$, and $e^{-2r} = (\sqrt{2}-1)/(\sqrt{2}+1)$ (7.7 dB squeezing). By using these tripartitely entangled modes, sender Alice can perform quantum telecloning of a coherent state input to two receivers Bob and Claire to produce Clone 1 and 2 at their sites. In other words, success of quantum telecloning is a sufficient condition for the existence of this type of entanglement.

For quantum telecloning, Alice first performs a joint measurement or so-called "Bell measurement" on her entangled mode ($\hat{x}_{\mathrm{A}}, \hat{p}_{\mathrm{A}}$) and an unknown input mode ($\hat{x}_{\mathrm{in}}, \hat{p}_{\mathrm{in}}$). In our experiment, the input state is a coherent state and a sideband of continuous wave 860 nm carrier light. The Bell measurement instrument consists of a 50/50 beam splitter and two homodyne detectors as shown in Fig. 1. Two outputs of the input 50/50 beam splitter are labeled as $\hat{x}_u = (\hat{x}_{\mathrm{in}} - \hat{x}_{\mathrm{A}})/\sqrt{2}$ and $\hat{p}_v = (\hat{p}_{\mathrm{in}} + \hat{p}_{\mathrm{A}})/\sqrt{2}$ for the relevant quadratures. Before Alice's measurement, the initial modes of Bob and Claire are, respectively,

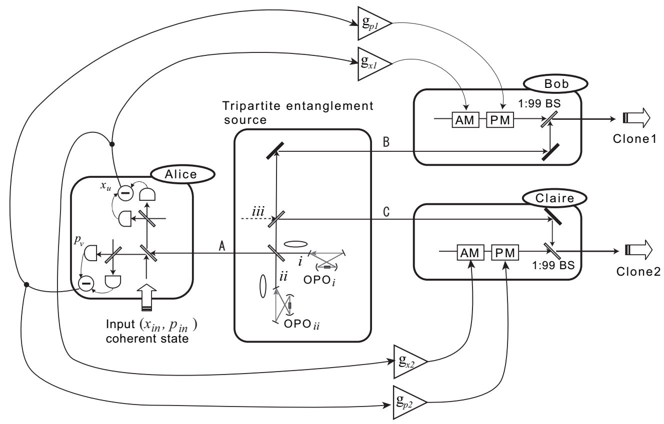

FIG. 1: The experimental set-up for quantum telecloning from Alice to Bob and Claire to produce Clone 1 and 2.

$$\hat{x}_{\mathrm{B,C}} = \hat{x}_{\mathrm{in}} - (\hat{x}_{\mathrm{A}} - \hat{x}_{\mathrm{B,C}}) - \sqrt{2}\hat{x}_u \qquad (2)$$

$$\hat{p}_{\mathrm{B,C}} = \hat{p}_{\mathrm{in}} + (\hat{p}_{\mathrm{A}} + \hat{p}_{\mathrm{B,C}}) - \sqrt{2}\hat{p}_v\ . \qquad (3)$$

Note that in this step Bob's and Claire's modes remain unchanged. After Alice's measurement on $\hat{x}_u$ and $\hat{p}_v$, these operators collapse and reduce to certain values. Receiving these measurement results from Alice, Bob and Claire displace their modes as $\hat{x}_{\mathrm{B,C}} \rightarrow \hat{x}_{1,2} = \hat{x}_{\mathrm{B,C}} + \sqrt{2}x_u$, $\hat{p}_{\mathrm{B,C}} \rightarrow \hat{p}_{1,2} = \hat{p}_{\mathrm{B,C}} + \sqrt{2}p_v$ and accomplish the telecloning. Note that the values of x_u and p_v are classical information and can be duplicated. In our experiment, displacement is performed by applying electro-optical modulations. Bob and Claire modulate beams by using amplitude and phase modulators (AM and PM in Fig. 1). The amplitude and phase modulations correspond to the displacement of p and x quadratures, respectively. The modulated beams are combined with Bob's and Claire's initial modes ($\hat{x}_{\mathrm{B,C}}, \hat{p}_{\mathrm{B,C}}$) at 1/99 beam splitters.

The output modes produced by the telecloning process are represented as [2],

$$\begin{aligned}\hat{x}_{1,2} &= \hat{x}_{\mathrm{in}} - (\hat{x}_{\mathrm{A}} - \hat{x}_{\mathrm{B,C}}) \\ &= \hat{x}_{\mathrm{in}} + \frac{1-\sqrt{2}}{2}\hat{x}_i - \frac{1+\sqrt{2}}{2}\hat{x}_{ii} \pm \frac{1}{\sqrt{2}}\hat{x}_{iii}^{(0)} \qquad (4)\\ \hat{p}_{1,2} &= \hat{p}_{\mathrm{in}} + (\hat{p}_{\mathrm{A}} + \hat{p}_{\mathrm{B,C}}) \\ &= \hat{p}_{\mathrm{in}} + \frac{1+\sqrt{2}}{2}\hat{p}_i - \frac{1-\sqrt{2}}{2}\hat{p}_{ii} \pm \frac{1}{\sqrt{2}}\hat{p}_{iii}^{(0)}\ , \qquad (5)\end{aligned}$$

where subscript iii denotes a vacuum input to the second beam splitter in the tripartite entanglement source, and + of $\pm$ for Clone 1 and $-$ for Clone 2. From these equations, we can see that the telecloned states have additional noise terms to the input mode $(\hat{x}_{\rm in}, \hat{p}_{\rm in})$. The additional noise can be minimized by tuning the squeezing levels of the two output modes of the OPOs. This corresponds to the minimization of the left-hand-side of Ineq. (1). In the ideal case with 7.7 dB squeezing, the additional noise is minimized and we obtain $\hat{x}_{1,2} = \hat{x}_{\rm in} - \frac{1}{2}(\hat{x}_i^{(0)} + \hat{x}_{ii}^{(0)}) \pm \frac{1}{\sqrt{2}}\hat{x}_{iii}^{(0)}$ and $\hat{p}_{1,2} = \hat{p}_{\rm in} + \frac{1}{2}(\hat{p}_i^{(0)} + \hat{p}_{ii}^{(0)}) \pm \frac{1}{\sqrt{2}}\hat{p}_{iii}^{(0)}$. These are the optimal clones of coherent state inputs [2]. In contrast to quantum teleportation, these optimal clones are degraded from the original input by one unit of vacuum noise. In the classical case, where no quantum entanglement is used, two units of vacuum noise would be added. This is dubbed the quduty which has to be paid for crossing the border between quantum and classical domains [21].

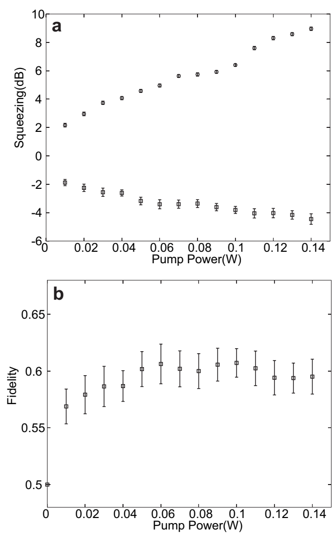

FIG. 2: (a) Pump power dependence of squeezing and antisqueezing of the output of OPO$_i$. The squeezing and antisqueezing are measured at 1 MHz. Visibility at a 50/50 beam splitter for homodyne measurement is about 0.95 and quantum efficiency of the detector is more than 99%. (b) Calculated fidelities from the squeezing and antisqueezing.

To evaluate the performance of telecloning, we use the fidelity $F = \langle\psi_{\rm in}|\hat{\rho}_{\rm out}|\psi_{\rm in}\rangle$ [22, 23]. The classical limit for coherent state cloning is derived by averaging the fidelity for a randomly chosen coherent input $F_{\rm av} = \frac{1}{2}$ [22, 24]. Experimentally, it is impossible to average over the entire phase space. However, if the gains of the classical channels $g_{x1,x2} = \langle\hat{x}_{1,2}\rangle/\langle\hat{x}_{\rm in}\rangle$ and $g_{p1,p2} = \langle\hat{p}_{1,2}\rangle/\langle\hat{p}_{\rm in}\rangle$ are unity $g_{x1,x2} = g_{p1,p2} = 1$, the averaged fidelity is identical to the fidelity for a particular coherent state input ($F_{\rm av} = F$). This is because the fidelity with unity gains is fully determined by the variances of the telecloned states, independent of the amplitude of the coherent state input. Experimental adjustment of $g_x = g_p = 1$ is performed in the manner of Ref. 10. The fidelity for a coherent state input with $g_x = g_p = 1$ can be written as [5],

$$F = 2/\sqrt{[1 + 4\langle(\Delta\hat{x}_{1,2})^2\rangle][1 + 4\langle(\Delta\hat{p}_{1,2})^2\rangle]}\ . \qquad (6)$$

From the above discussion, if we measure $\langle(\Delta\hat{x}_{1,2})^2\rangle$ and $\langle(\Delta\hat{p}_{1,2})^2\rangle$ of the outputs for a coherent state input and get $F > \frac{1}{2}$, then the quantum telecloning of coherent states is deemed successful. Note that the optimal fidelity of Gaussian coherent-state telecloning [2] is $\frac{2}{3}$, which is consistent with the parameters of the ideal case mentioned above (see Ref. 25 for a non-Gaussian result).

Fig. 2a shows the typical pump power dependence of squeezing and antisqueezing of the output of the OPOs. Here the OPO cavities contain Potassium Niobate crystals inside as nonlinear mediums and are pumped with the frequency doubled outputs of a continuous wave Ti:sapphire laser at 860 nm. In order to minimize the asymmetry of squeezing without sacrificing the level of squeezing, we select mirrors with reflectivity of 12% for the output couplers of the OPOs. With Eqs. (4), (5) and (6) and these experimental results, we calculate the expected fidelities of the telecloning experiments, which are plotted in Fig. 2b. Accordingly, we set the pump power to 60 mW for which we expect the fidelity to be $\simeq 0.6$.

Quantum telecloning was performed for two types of input states: a vacuum state and a coherent state that is created by applying electro-optic modulation to a very weak carrier beam. Fig. 3 summarizes the results from both experiments, with Alice's states in Fig. 3a, and Bob and Claire's output states in Fig. 3b and c. Trace ii of Fig. 3a shows Alice's p-quadrature measurement for a vacuum input, $\langle(\hat{p}'_v)^2\rangle$, where $\hat{p}'_v = (\hat{p}_{\rm in}^{(0)} + \hat{p}_{\rm A})/\sqrt{2}$. Note that $\langle\hat{p}_{\rm in}^{(0)}\rangle = \langle\hat{p}_{\rm A}\rangle = 0$; thus $\langle(\hat{p}'_v)^2\rangle = \langle(\Delta\hat{p}'_v)^2\rangle = \langle(\Delta\hat{p}_v)^2\rangle$, because the vacuum is a zero-amplitude coherent state. The noise level is 2.1 dB higher compared to the vacuum noise level $\langle(\Delta\hat{p}^{(0)})^2\rangle = \frac{1}{4}$, due to the "entangled noise" $\hat{p}_{\rm A}$. This noise is canceled to some extent by the tripartite entanglement. Trace iii in Fig. 3a shows Alice's coherent-state input with the phase scanned. Consistent with the above discussion on the variance of a coherent state input, the troughs of trace iii are level with trace ii within experimental accuracy. Note that Alice's 50/50 beam splitter reduces the amplitude of the measured state (i.e., the peaks of trace iii) by 3 dB relative to the input state.

Figs. 3b and c show the measurement results of the telecloned states. Traces ii show the results for a vacuum input, $\langle(\Delta\hat{p}_{1,2})^2\rangle$. The noise level for Clone 1 is 4.06 ± 0.17 dB and that for Clone 2 is 4.03 ± 0.15 dB. We also measured the x-quadratures $\langle(\Delta\hat{x}_{1,2})^2\rangle$ and obtained 3.74 ± 0.15 dB for Clone 1 and 3.79 ± 0.15 dB for Clone 2 (not shown). Note that the telecloned states

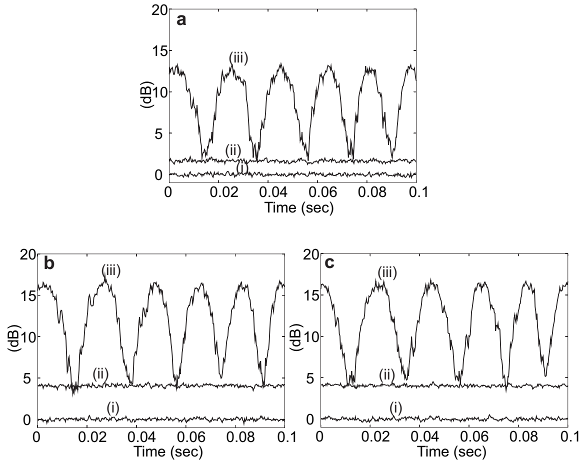

FIG. 3: Quantum telecloning from Alice to Bob and Claire. All traces are normalized to the corresponding vacuum noise levels. (a) Alice's measurement results for p quadrature. Trace i, the corresponding vacuum noise level $\langle(\Delta\hat{p}^{(0)})^2\rangle = \frac{1}{4}$. Trace ii, the measurement result of a vacuum input $\langle(\hat{p}'_v)^2\rangle$ where $\hat{p}'_v = (\hat{p}^{(0)}_{\rm in} + \hat{p}_{\rm A})/\sqrt{2}$. Trace iii, the measurement result of a coherent state input $\langle(\hat{p}_v)^2\rangle$ with the phase scanned. (b),(c) The measurement results of the telecloned states at Bob (b) and Claire (c) for p quadratures (x quadratures are not shown). Trace i, the corresponding vacuum noise levels. Trace ii, the telecloned states for a vacuum input $\langle(\Delta\hat{p}_{1,2})^2\rangle$. Trace iii, the telecloned states for a coherent state input. The measurement frequency is centered at 1 MHz, and the resolution and video bandwidths are 30 kHz and 300 Hz, respectively. All traces except for trace iii are averaged 20 times.

have the same mean amplitude as that of the input inferred from Alice's measurement, which is consistent with the unit gains of the classical channels. Finally, we calculated the fidelity from Eq. (6), and found $F = 0.58 \pm 0.01$ for both teleclones. Since this fidelity exceeds the classical cloning limit of $\frac{1}{2}$, we have successfully demonstrated $1 \rightarrow 2$ quantum telecloning of coherent states. These results are operational evidence for the existence of the tripartite entanglement. The slight discrepancies from the expected fidelities are attributed to fluctuations of phase locking of the system.

We note that, in quantum cryptographic scenarios, quantum telecloning, complemented by quantum storage, may provide a means for an eavesdropper to remotely monitor a quantum cryptographic channel more securely. In particular, the remote nature of the operation offers the advantage that even if eavesdropping is discovered, her identity and location are guaranteed uncompromised. As distinct from the symmetric telecloning reported here, asymmetric telecloning might be the method of choice by a technologically advanced eavesdropper. This can easily be achieved in our scheme by modifying the shared state and feedforward gains. In addition, to make the scheme practical, high capacity quantum memory would be essential for the multiply entangled states used to operate the protocol autonomously. This would guarantee the eavesdropper's anonymity and the flexibility to perform individual, collective or coherent attacks without the need for backward communication. Promising strides in the storage of continuous-variables states [26, 27] would also give suitable storage for entangled states. Thus, the successful demonstration of coherent-state telecloning offers a step forward in the technological toolkit of eavesdroppers of quantum cryptographic channels.

In conclusion, we have demonstrated $1 \rightarrow 2$ quantum telecloning of coherent states for continuous variables. Manipulations of multipartite entanglement are essential for realization of quantum computation and quantum communication among many parties. In particular, this paper reported a demonstration of manipulations of a new type of multipartite entanglement and an example of the reduction of the number of steps in quantum information processing with entanglement. The techniques used in this experiment are easily extendable to $1 \rightarrow n$ quantum telecloning and related operations.

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